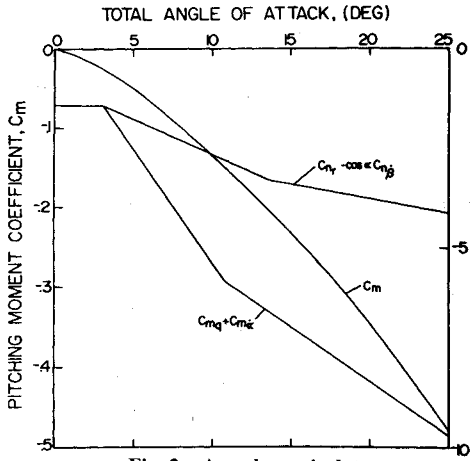

Fig. 2 Aerodynamic data.

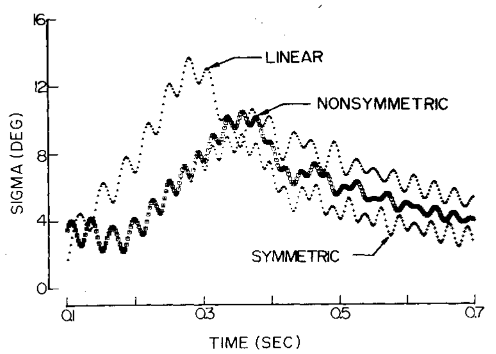

Fig. 3 Resultant angles of attack, $\delta_{t_0}=2.15°$.

Table 1 Maximum resultant angle of attack at resonance

δ_{t_0}	σ_{max} (Deg)		
(Deg)	Linear	Symmetric	Nonsymmetric
1.10	7.02	5.66	5.85
2.15	13.69	9.20	10.21
3.53	17.46	11.80	14.09

These equations of motion were numerically integrated on a computer to determine the resultant angle-of-attack σ as a function of time. Typical mass parameters for a slender, symmetric, re-entry vehicle were employed. The $\sigma(t)$ was computed by giving the re-entry vehicle an assumed constant roll acceleration p which carried it through resonance. The computations were terminated when the roll rate equaled five-times the pitching frequency. Constant values of altitude and velocity were chosen so that the aerodynamic coefficients reported in Ref. 8 (Fig. 2) were applicable. The aerodynamic asymmetry coefficients C_{m_ϵ} were obtained for three typical values of the nonrolling trim angle, $\delta_{t_0}=1.10$, 2.15, and 3.53°.

To distinguish between the different sets of stability coefficients, the following definitions were employed

"Linear" means

$$C_m(\sigma)=(\partial C_m/\partial\sigma)_{\sigma=0}\sigma;C_{m_q}+C_{m_{\dot\alpha}}=C_{n_r}-\cos\alpha C_{n_{\dot\beta}}$$

"Symmetrical" means

$$C_m \quad =f(\sigma),\ C_{m_q}+C_{m_{\dot\alpha}}=C_{n_r}-\cos\alpha C_{n_{\dot\beta}}=g(\sigma)$$

"Nonsymmetrical" means

$$C_m \quad =f(\sigma);\ C_{m_q}+C_{m_{\dot\alpha}}=g(\sigma),\ C_{n_r}-\cos\alpha C_{n_{\dot\beta}}=h(\sigma)$$

Results

A typical set of resultant angles of attack as functions of time is presented in Fig. 3. The maximum values of the resultant angles of attack at resonance are given in Table 1. It is seen that linear stability coefficients overpredict the trim magnification at resonance. More importantly, symmetric stability coefficients, with the incorrect assumption that $C_{m_q}+C_{m_{\dot\alpha}}=C_{n_r}-\cos\alpha C_{n_{\dot\beta}}=g(\sigma)$, underestimates the trim magnification at resonance and leads to overly optimistic stability predictions. In view of the current interest in high angle-of-attack maneuvering vehicles, emphasis should be placed on the measurement of nonsymmetric stability coefficients and the evaluation of their effects on stability and performance.

References

[1] Nicolaides, J. D., "Missile Flight and Astrodynamics," TN 100A, 1959-61, Bureau of Weapons, Dept. of the Navy, Washington, D.C.

[2] Murphy, C. H., "An Erroneous Concept Concerning Nonlinear Aerodynamic Damping," *AIAA Journal*, Vol. 1, June 1963, pp. 1418-1419.

[3] Tobak, M., Schiff, L. B., and Peterson, V. L., "Aerodynamics of Bodies of Revolution in Coning Motion," *AIAA Journal*, Vol. 7, Jan. 1969, pp. 95-99.

[4] Schiff, L. B. and Tobak, M., "Results from a New Wind Tunnel Apparatus for Studying Coning and Spinning Motions of Bodies of Revolution," *AIAA Journal*, Vol. 8, Nov. 1970, pp. 1953-1957.

[5] Stone, G. W., Clark, E. L. and Burt, G. E., "An Investigation of Nonsymmetric Aerodynamic Damping Moments," AIAA Paper 72-29, San Diego, Calif., 1972.

[6] Walchner, O. and Sawyer, F. M., "In-Plane and Out-of-Plane Stability Derivatives of a Slender Cone at Mach 14," ARL 73-0090, July 1973, Aerospace Research Laboratories, Wright-Patterson AFB, Ohio.

[7] Tobak, M. and Schiff, L. B., "A Nonlinear Aerodynamic Moment Formulation and its Implications for Dynamic Stability Testing," AIAA Paper 71-275, Albuquerque, New Mexico, 1971.

[8] Sawyer, F. M., Ingram, C. W., "In-Plane and Out-of-Plane Stability Derivatives of a Slender Biconic Reentry Vehicle at Mach 14," ARL TR 74-0135, Nov. 1974, Aerospace Research Laboratories, Wright-Patterson, AFB, Ohio.

Circular Earth Orbits Attainable with Fixed Two-Impulse Expendable Tug

Richard F. Porter*

Battelle Columbus Laboratories, Columbus, Ohio

Introduction

ALTHOUGH solid rocket motors in expendable stages have been suggested as an alternative concept for Shuttle interim upper stage applications, a lack of mission flexibility is sometimes cited as a disadvantage of these fixed-impulse systems. Specifically, the sizes of the stages are dictated by the requirements of a specific mission, typically the delivery of a given payload to geosynchronous orbit. For other payloads and orbits, it is generally necessary to perform nonoptimal maneuvers.

This paper describes a limited assessment of the performance penalties inherent with fixed-impulse motors from the standpoint of mission flexibility. Only nonreturn missions to circular Earth orbits are considered. It is assumed that the

Received May 27, 1975. This work was conducted for the NASA Office of Space Science, Washington, D.C., under Contract NASw-2018.

Index categories: LV/M Mission Studies and Economics; LV/M Trajectories; Solid and Hybrid Rocket Engines.

*Senior Researcher. Member AIAA.

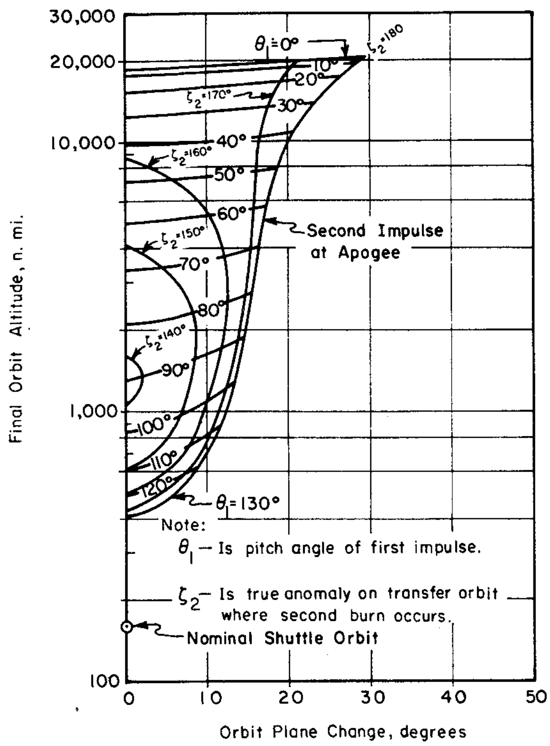

Fig. 1 Attainable orbits with zero yaw on first burn.

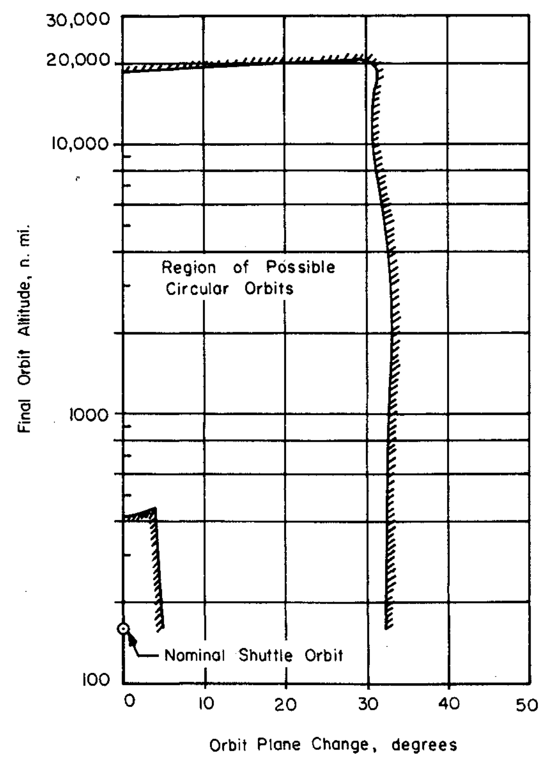

Fig. 2 Attainable circular orbits with fixed two-impulse tug (geosynchronous design payload).

system consists of two stages and payload, released from a circular Shuttle orbit at 160 naut miles altitude. It is further assumed that the stages have been sized to deliver a given payload mass to synchronous equatorial orbit.

The investigation addresses two questions: 1) With the design payload mass, what range of circular orbit altitudes and inclination changes (with respect to the Shuttle orbit) is attainable? 2) How does this performance compare with that achievable if the two impulses could be varied, subject only to the constraint that the sum be identical to the fixed-impulse system (i.e., if the energy were manageable)?

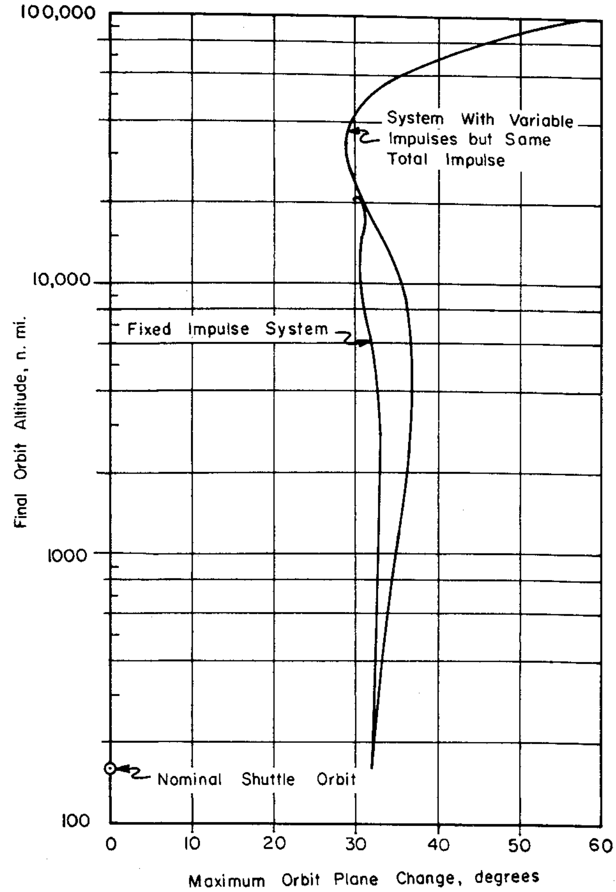

Fig. 3 Comparison of fixed and variable impulse systems.

Circular Orbits Attainable with Design Payload

For the first part of this investigation, the payload was fixed at the design value for the geosynchronous mission, assuming launch from a 160 naut miles altitude, 28.5° inclination Shuttle orbit. With the payload held constant, the velocity changes are also constant in magnitude for each burn, at 8100 fps and 6000 fps, respectively.

Two parameters are necessary to specify the direction of the first impulse; these were chosen as the pitch and yaw angles relative to the Shuttle velocity vector. Each pair of angles then uniquely defines a transfer orbit characterized by a new apogee, perigee, and plane change with respect to the Shuttle orbit.

A parametric sweep was made with small increments of both pitch and yaw to compute the set of possible transfer orbits having an apogee greater than 160 naut miles. Then, at several discrete points on each transfer orbit (increments of true anomaly), the possibility of obtaining a circular orbit with the remaining velocity impulse of 6000 fps was examined.

For a final circular orbit, the second burn must always satisfy two constraints: 1) the vertical component of the second impulse must be equal and opposite to the vertical velocity on the transfer orbit, and 2) the magnitude of the total velocity after firing must be equal to the circular orbit speed at the altitude in question. These conditions completely specify the pitch angle required for the second burn, but the yaw angle is defined only in magnitude and can be directed toward either side of the transfer orbit plane to either maximize or minimize the total plane change.

A map of possible circular orbits, for the special case of zero yaw angle at first burn, is given in Fig. 1. The parameters shown are the pitch angle at first burn (relative to horizontal) and the location on the transfer orbit at which the second burn occurs, as defined by the true anomaly. For Fig. 1, the direction of the yaw angle at second burn was selected to maximize the plane change.

Two features of Fig. 1 are noteworthy. First, the greatest orbit altitude and plane change occur when both burns are horizontal, as represented by the point computed for zero initial pitch angle and true anomaly of 180° (apogee) for the second burn. This point is the design geosynchronous mission. Secondly, circular orbits coplanar with the Shuttle can be obtained over a very wide range of altitudes from about 400 to almost 20,000 naut miles.

If the yaw angle at first burn is chosen to be other than zero, the map of possible circular orbits is generally similar, in that the maximum attainable altitude and the maximum plane change always occur simultaneously when both burns are horizontal. On the other hand, if an orbit coplanar with the Shuttle orbit is desired, it is necessary to perform the second burn on the opposite side of the Earth from the first, and to achieve a second plane change equal in magnitude to the first. These conditions preclude the possibility of coplanar orbits at altitudes below about 2200 naut miles, if the initial burn is out-of-plane.

With all parameters free, the envelope of the entire region of attainable circular orbits is shown in Fig. 2, for the design payload. At this point, it should be emphasized that plane changes with respect to the Shuttle orbit (as shown in Fig. 1 and 2) are not necessarily synonomous with inclination changes with respect to the equatorial plane. In fact, for a given plane change, the resulting inclination change is a sinusoidal function of the position on the Shuttle orbit where the maneuver begins. The maximum inclination change is equal to the orbit plane change, but zero inclination change is also obtainable for any value of plane change.

Consequently, from Fig. 2, we may conclude that the fixed two-impulse system can deliver the design payload mass to any circular orbit up to synchronous altitude and to any inclination within approximately 30° of the Shuttle orbit.

Disadvantages of Fixed Impulse Ratio

To answer the second question, computations were made of the circular orbits which could be attained if only the total impulse were fixed, and could be distributed in any ratio between the two burns. The comparison is shown in Fig. 3, which illustrates that the primary limitation of the fixed impulse system is an inability to achieve circular orbits higher than geosynchronous altitude. At all lower altitudes, the advantage of the variable impulse system is limited to a slightly larger plane change capability.

Conclusions

From this investigation, the following conclusions are drawn: 1) A fixed two-impulse system, designed to place a given payload into synchronous equatorial orbit from the nominal Shuttle orbit, can deliver the same payload mass to any lower circular orbit at any inclination within approximately 30° of the Shuttle orbit. 2) If the fixed two-impulse system is compared to a variable two-impulse system with the same total impulse, the only clear disadvantage of the fixed impulse system is an inability to achieve circular orbits at very high altitudes (above geosynchronous altitudes with the design payload).